Liquid Holdup and Heat Transfer Coefficient in Liquid-Solid and Three-Phase Fluidized Beds

Liquid holdups and wall-to-bed heat transfer coefficients in three-phase fluidized beds with glass beads and cylindrical γ-alumina particles were determined experimentally. It was found that the product of heat transfer coefficient and liquid holdup in three-phase fludized beds is equal to that in corresponding liquid-solid fluidized beds. A generalized correlation equation for heat transfer coefficients in both liquid-solid and three-phase fluidized beds was proposed based on the above relationship.

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SCOPE

This study investigates the relationships between liquid holdup and heat transfer coefficient in three-phase fludized beds and proposes a correlation equation which can be used for both liquid-solid and three-phase fluidized beds. Both liquid-solid and gas-liquid-solid fluidized beds are used in the chemical industry. The former are used for crystallization, ion exchange, hydrometallurgical operations, and so on. A three-phase fluidized bed as defined in the current study is a bed of particles fluidized by the cocurrent upward flow of gas and liquid. Both liquid and gas flow through the bed and maintain the expanded bed of solid particles in a continuous random motion. The liquid phase forms a continuous medium, and the gas is dispersed as discrete bubbles. One of the most important features of a fluidized bed is its good heat transfer characteristics. Noteworthy applications of three-phase fluidized beds with cocurrent upward flow of gas and liquid are in catalytic reactors, gas absorbers, and fermenters. In these applications the addition or removal of heat through the retaining walls of the bed or immersed surface is often of prime importance. A knowledge of the value of wall-to-bed heat transfer coefficient is essential for the estimation of heat transfer area.

Heat transfer in liquid-solid fluidized beds has been extensively studied and substantial experimental data have been accumulated. Although several correlating equations are reported in the literature, their accuracy is limited to a narrow range of published data. It is only in recent years that a sufficient number of papers have become available on the topic of heat transfer in three-phase fluidized beds to attempt generalized correlations.

Phase holdup is one of the important hydrodynamic phenomena in three-phase fluidized beds. It affects the bed volume and the residence time of the fluid phase. In particular, the liquid holdup plays an important role in correlating heat transfer coefficients (Kato et al. 1981; Chiu and Ziegler, 1983).

CONCLUSIONS AND SIGNIFICANCE

Liquid holdup was found to decrease with increasing gas velocity and decreasing liquid velocity for all particle sizes studied. Gas and solid holdups, on the other hand, exhibit changes which depend on particle size. Heat transfer coefficient increases with gas velocity for all particle sizes studied in three-phase fluidized beds. The product of heat transfer coefficient and liquid holdup in liquid-solid fluidized beds is equal to that in three-phase fluidized beds. The heat transfer coefficients in both liquid-solid and three-phase fluidized beds can be correlated by $St_{m3} = St_{m2} = 0.127\ Re_{m2}^{-0.354} Pr^{-0.362} u_R^{0.266} \phi_s^{-1}$.

The above correlation covers a wide range of particle size, particle density, particle shape, column diameter, gas and liquid velocities, and liquid viscosity. It should find use in heat transfer equipment and reactor design.

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BACKGROUND

Some design aspects of liquid-solid fluidized beds have been reviewed recently by Josi (1983).

Mathematical models of hydrodynamics in three-phase fluidized beds have been proposed by ØStergaard (1965), Bhatia and Epstein (1974), Darton and Harrison (1975), Dhanuka and Stepanek (1978), and recently by Khang et al. (1983). To apply those models it is necessary to know either bubble rising velocity or gas holdup data. Kato et al. (1981) correlated the liquid holdup in three-phase fludiized beds by modifying the equation of Garside and Al-Dibouni (1977). The present authors have reported that the relative increase in heat transfer coefficient is equal to the relative decrease in liquid holdup in a three-phase fluidized bed (Chiu and Ziegler, 1983).

Kato et al. (1981) have proposed a correlation equation for both liquid-solid and three-phase fluidized beds based on the results of relatively small particles fluidized by water and CMC solution with air. A correlation equation for large particles fluidized by water and air was presented by the present authors (Chiu and Ziegler, 1983). A summary of experimental conditions on previous heat transfer investigations in three-phase fluidized beds was reported in that study.

EXPERIMENTAL

Hydrodynamic experiments were carried out in a cylindrical Lucite column of 0.0508 m dia. with ten pressure taps at 0.127 m intervals. The column was 1.524 m long, with the first tap 0.1524 m above the distributor. The top of the column protruded into a concentrically-mounted outlet header acting as a weir over which liquid flows. This maintained an approximately constant head in the column. The calming section, 0.3048 m long, was composed of two compartments. The lower compartment contained a bundle of 0.00953 m O.D. polyethylene tubing, 0.152 m long. The upper compartment was filled with a 0.0762 m deep bed of 0.003 m dia. glass beads. On top of the beads a thin ring with attached 0.00015 m and 0.00025 m clear opening stainless steel screens was placed between flanges to form the bed support. The beads, bed, and tube bundle were each supported with 0.00059 m clear opening brass screen similarly installed. The flow diagram is presented in Figure 1.

Solid particles were glass beads of 0.001 m and 0.003 m dia., and cylindrical γ-alumina of [dia. (m) $\times$ length (m)] 0.003175 $\times$ 0.003175, 0.00476 $\times$ 0.003175, and 0.00476 $\times$ 0.00476. The properties of solid particles used have been previously described (Chiu and Ziegler, 1983). Air was used as the gas phase, and tap water was used as the liquid phase.

Liquid from the storage tank was pumped to the column and passed through one of the rotameters and a valve on the bypass connecting the pump discharge to the storage tank. Prior to entering the column, laboratory air passed through one of the rotameters, a gas meter, and a distributor below the calming section. Both liquid and gas were passed through the calming section. When a steady state was reached, the dynamic pressure-drop profile along the column was measured using a carbon tetrachloride manometer. The bed height was taken as the intersection of the pressure-drop profile curves on the plot of pressure drop vs. column height. The phase holdups were determined from the slope of the axial profiles of dynamic pressure drop, bed height, and properties of liquid and solids. Further details may be found in the work of Chiu (1982).

Heat transfer experiments were carried out in a copper column of 0.0508 m dia. The column was heated with electric heaters at constant heat flux. To avoid including any thermal entrance length effects, the difference between measured wall temperature and bulk stream temperature of the bed at the level of the main heater was used rather than an integrated or logarithmic mean temperature difference to calculate the heat transer coefficient. The details of the experimental setup and data treatment are discussed elsewhere (Chiu and Ziegler, 1983).

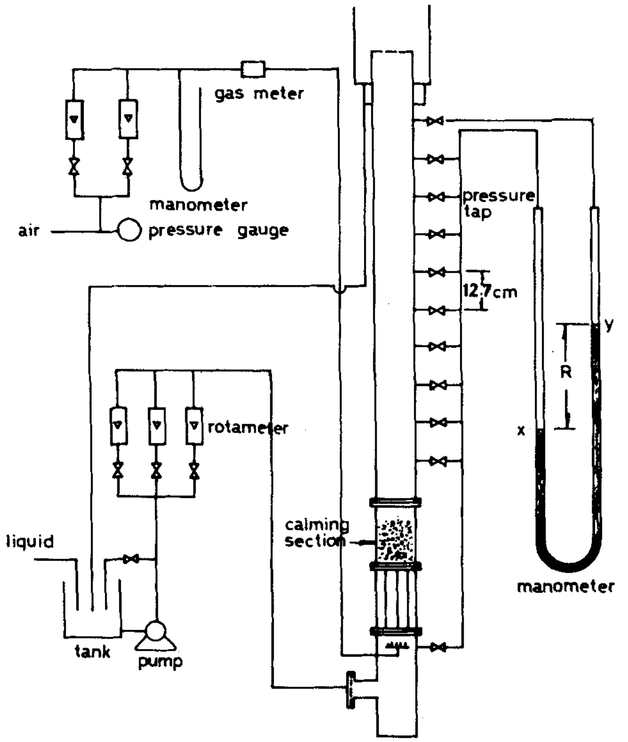

Figure 1. Experimental apparatus for holdup measurements.

RESULTS AND DISCUSSION

Liquid Holdup

Extensive data on the individual phase holdups, in particular, for spherical particle systems have been reported in the literature. The present experimental results show that liquid holdup decreases with increasing gas velocity and decreasing liquid velocity for all particle sizes studied. The behavior of liquid holdup is unlike gas and solid holdups which exhibit different behavior for different particle size. Kato et al. (1981) proposed the following equations to correlate the liquid holdup in three-phase fluidized beds.

$$\epsilon_{L3} = \epsilon_L^* \left(\frac{u_L}{u_T}\right)^{1/n} \tag{1}$$

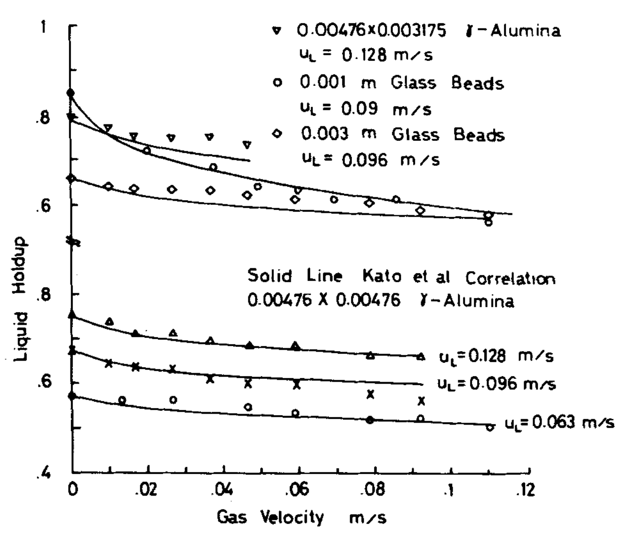

Figure 2. Representative liquid holdup variations in three-phase fluidized beds with gas and liquid velocity.

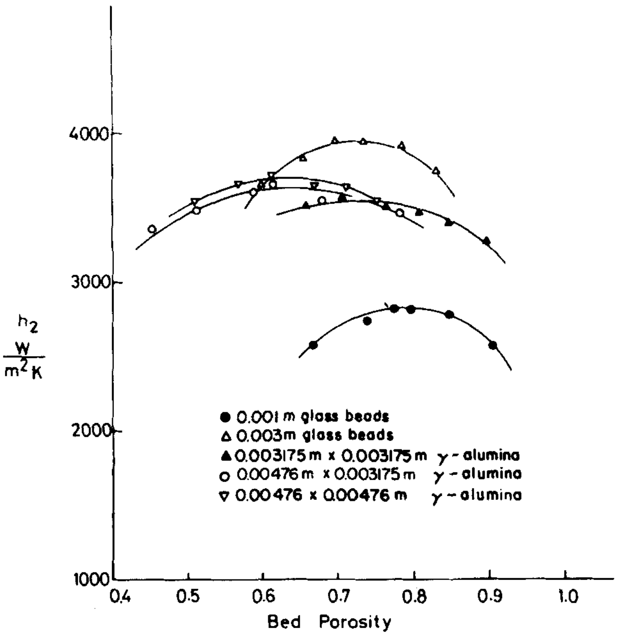

Figure 3. Effect of bed porosity on the heat transfer coefficient in liquid-solid fluidized beds.

where ϵ_L^* is the extrapolated value of the liquid holdup at $u_L = u_T$. It can be correlated as

$$\epsilon_L^* = 1 - 9.7(350 + Re_T^{1.1})^{-0.5}K^{0.092} \tag{2}$$

where K is defined by

$$K = \frac{\rho_L u_G^4}{g\sigma}$$

The value of n can be calculated by the following equation

$$\frac{5.1(1 + 16.9K^{0.285}) - n}{n - 2.7} = 0.1(1 + 4.43K^{0.165})Re_T^{0.9} \tag{3}$$

Liquid holdups observed in three-phase fluidized beds by Chiu (1982) were compared with Kato's correlation. Some results are shown in Fig. 2. It can be seen that the liquid holdup values observed in the present work are in agreement with Kato's correlation.

Heat transfer in Liquid-Solid Fluidized Beds

Figure 3 presents heat transfer coefficients vs. bed porosity data for the five particle types. It can be seen that heat transfer coefficients pass through a maxima as bed porosity is progressively increased. In addition, the bed porosity at the maximum value decreases with increasing particle size. It is also found that the maximum heat transfer coefficient value increases with increasing particle size for a given particle material. This might suggest that heat transfer via particle carrier is not the main path for heat transfer in liquid-solid fluidized beds. When heat transport occurs via a particle carrier mode, an inverse relationship between heat transfer coefficient and particle size is expected, since the smaller the particles the greater the number per unit wall surface area, and consequently the greater the heat transferred by their turnover.

Wall-to-bed heat transport in a liquid-solid fluidized bed is thought to occur through two zones in series; i.e., total resistance consists of a wall resistance and bed resistance, as per Wasmund and Smith (1967), and Simpson (1973). The shift of the controlling thermal resistance from the wall to bed occurs with a progressive decrease in either particle size or superficial liquid velocity. The maximum heat transfer coefficient observed in Figure 2 might result from the competitive influence of a situation which increases wall resistance and decreases bed resistance simultaneously.

Heat Transfer in Three-Phase Fluidized Beds

The details of the experimental result and discussion have been reported previously (Chiu and Ziegler, 1983). In the previous study, it was found that heat transfer coefficients in three-phase fluidized beds increase with gas velocity at a given superficial liquid velocity and particle size. Heat transfer coefficients in three-phase fluidized beds are higher than those of comparable gas-liquid systems except for small spherical particle size situations at high gas velocity. Plots of heat transfer coefficient vs. bed porosity or liquid velocity pass through a maximum as either the bed porosity or liquid velocity is progressively increased. The bed porosity at which the maximum value occurs decreases with increasing particle size.

DATA CORRELATION

Richardson (1971) suggested that a modified Reynolds number be used to correlate pressure drop data. The modified Reynolds number had previously been used to correlate heat transfer data by Hamilton (1970) and was used later by Richardson et al. (1976). In the modified Reynolds number, velocity is taken as the velocity of fluid relative to the particles, and the linear dimension of characteristic length is expressed as the volume of pore space per unit surface of particles. Therefore, the modified Reynolds number for liquid-solid fluidized beds can be expressed as

$$Re_{m2} = \frac{u_L \rho_L}{S(1 - \epsilon_{L2})\mu_L} \tag{4}$$

where S is the surface area of particle per unit volume and $S = 6/d_p$ for spherical particles or $S = 2(2L + d_p)/d_pL$ for cylindrical particles.

It is known that liquid holdup is changed when gas is introduced to a liquid-solid fluidized bed. For three-phase fluidized beds, characteristic velocity is given by u_L/ϵ_{L3}, and the characteristic length is expressed as $\epsilon_3/S(1 - \epsilon_3)$. Then the modified Reynolds number for three-phase fluidized beds can be expressed as

$$Re_{m3} = \frac{u_L \rho_L \epsilon_3}{S(1 - \epsilon_3)\epsilon_{L3}\mu} \tag{5}$$

Similarly, the modified Stanton number is given by

$$St_{m2} = \frac{h_2 \epsilon_{L2}}{\rho_L C_{pL} u_L} \tag{6}$$

for liquid-solid fluidized beds, and

$$St_{m3} = \frac{h_3 \epsilon_{L3}}{\rho_L C_{pL} u_L} \tag{7}$$

for three-phase fluidized beds.

Heat Transfer in Liquid-Solid Fluidized Beds

In the present study, the following dimensionless groups are proposed to correlate the heat transfer coefficients in liquid-solid fluidized beds.

$$St_{m2} = f(Re_{m2}, Pr, u_R, \phi_s) \tag{8}$$

Where u_R is the ratio of minimum fluidization velocity in liquid-solid fluidized beds to terminal velocity, and ϕ_s is shape factor.

The effect of liquid viscosity on the heat transfer coefficients in liquid-solid fluidized beds has been studied by Romani and Richardson (1974), Richardson et al. (1976), Kato et al. (1981), and Kang et al. (1983). The test fluid of the first two reports was dimethyl

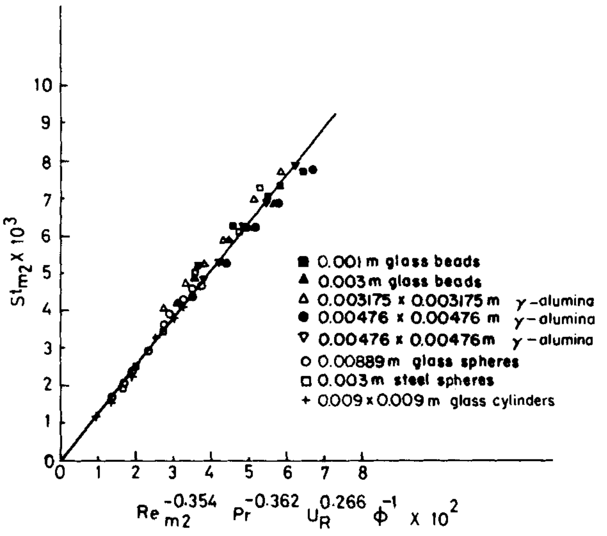

Figure 4. Correlation for heat transfer coefficients in liquid-solid fluidized beds.

phthalate; CMC solution was used as test fluid by Kato et al. and Kang et al. A multiple linear regression method was employed to correlate the present data and those of Richardson et al. (1976) with Eq. 8. The latter data were used for considering the effect of liquid viscosity on the heat transfer coefficients. The following correlation equation was obtained with a 0.984 Pearson correlation coefficient.

$$St_{m2} = 0.127\, Re_{m2}^{-0.354} Pr^{-0.362}\, u_R^{0.266} \phi_s^{-1} \tag{9}$$

The validity of Eq. 9 may be inferred from Figure 4. The average error of the present correlation is within ±10%.

Equation 9 can be rearranged for spherical particles, noting that:

$$\frac{h_2}{C_{PL}\rho_L u_L} = \frac{h_2 d_p}{k}\left(\frac{u_L d_p \rho_L}{\mu_L}\right)^{-1}\left(\frac{C_{pL}\mu_L}{k}\right)^{-1} \tag{10}$$

and

$$S = \frac{6}{d_p} \tag{11}$$

Table 1. Comparison of Published Data with Correlations by Present Study and by Kato of Heat Transfer Coefficients in Liquid-Solid Fluidized Beds

Published Data Source	Kato's* Correlation	Present Correlation	Test Fluid
Baker et al. (1978)	5.9%	5.42%	Water
Kang et al. (1983)	5.65%	3.9%	Water
Kang et al. (1983)	30.9%	17.2%	CMC solution
Richardson et al. (1976)	39.9%	3.9%	Dimethyl phthalate
Wasmund and Smith (1967)	13.5%	4.41%	Water

* Kato et al. (1981).

to give

$$Nu_2 = \frac{h_2 d_p}{k} = 0.762\, Re_{m2}^{0.646}\, Pr^{0.638} u_R^{0.266} \phi_s^{-1} \frac{1-\epsilon_{L2}}{\epsilon_{L2}} \tag{12}$$

Kato et al. (1981) proposed an equation to correlate heat transfer coefficient based on the results of relatively small size glass beads fluidized by water and CMC solution. A comparison of the average absolute error of the present correlation equation with that of Kato et al. for the results of Richardson et al. (1976), Baker et al. (1978), Kang et al. (1983), and Wasmund and Smith (1967) is shown in Table 1. The average absolute error defined in the current study is the ratio of the difference between the calculated values of Nu_2 and those experimentally observed to the experimental values. A summary of the various experimental conditions is presented in Table 2.

The larger average absolute error for Kang's work with CMC solution may be due to the pseudoplastic fluid behavior of CMC solution. It should be noted that the present correlation equation is not expected to apply to bed porosities greater than 0.85 at high viscosities (>10 cp = Pa·s), because the modified Reynolds number Re_{m2} is not usually found to be relevant at high bed porosities.

The data of Wasmund and Smith for Aluminum spheres are well estimated with the present correlation equation. It is well known that the thermal conductivitiy of aluminum particles is much higher than that of glass beads (relative thermal conductivity about 250). However, there is no term which includes solid thermal conductivity in the present correlation equation. Therefore,

Table 2. Summary of Various Authors' Experimental Conditions

Authors	Column dia. m	U_L m/s	Particle dia. m	
Baker et al. (1978)	0.24	0.04–0.126	0–0.238	0.001, 0.003, 0.005 glass beads
Kang et al. (1983)	0.152	0.002–0.16	0–0.12	0.0017–0.008 glass beads
Kato et al. (1981)	0.052 0.12	0.005–0.11	0–0.1	0.00042–0.0022 glass beads
Richardson (1971, 1976)	0.102	0.047–0.282	—	0.009 glass spheres 0.009 × 0.009 glass cylinders 0.003 steel spheres
Wasmund & Smith (1967)	0.0523	0.015–0.225	—	0.000635–0.001981 aluminum
Present work	0.0508	0.063–0.15	0–0.137	0.001–0.003 glass beads 0.003175× 0.003175, 0.00476× 0.003175, 0.00476× 0.00476 γ-alumina

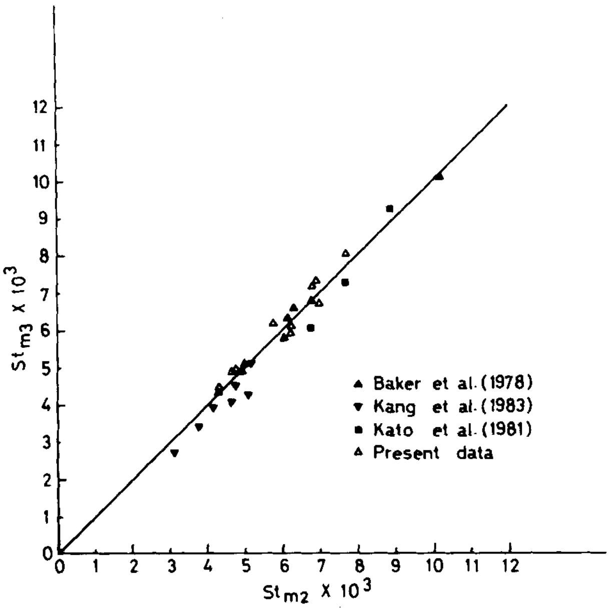

Figure 5. The relationship between St_{m2} and St_{m3}.

wall-to-bed heat transfer coefficients in liquid-solid fluidized beds probably are independent of solid thermal conductivity. Further, the present correlation equation predicts that heat transfer coefficients of a system with spherical particles are slightly less than those of cylindrical particle systems of the same material with the same effective diameter. This trend was previously observed by Richardson et al. (1976), and may be due to sharp edges causing greater attrition of the boundary layer in the case of cylindrical particles.

The data examined in the present work cover column diameters in the range 0.0508 m to 0.24 m. The effect of column diameter on heat transfer coefficient does not appear explicitly in Eqs. 9 and 14. However, ϵ_{L2} is influenced by column diameter via wall effects. The wall effect on terminal velocity is slight for systems of relatively small particles. The effect of column diameter on heat transfer coefficient should therefore be slight in the case of systems with relatively small particles. However, Kato's results for heat transfer coefficient observed in a column of 0.052 m dia. are about 30% less than those observed in a 0.12 m I.D. column with relatively small glass beads. The reason is not clear.

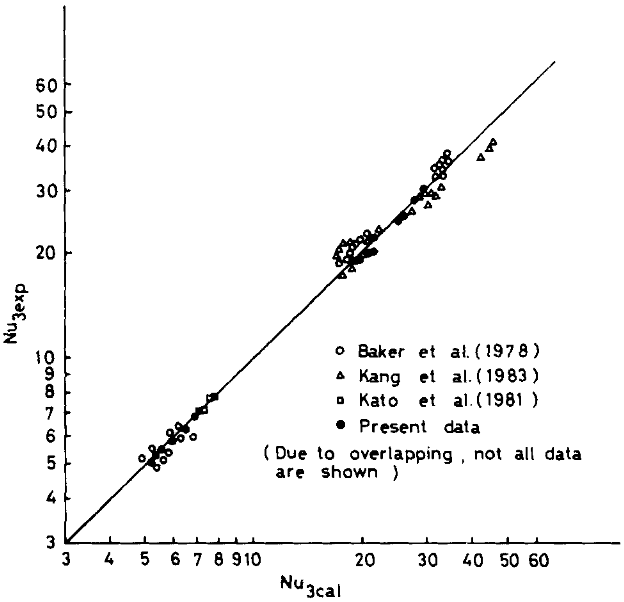

Figure 6. Comparison of Nu_{3exp} with Nu_{3cal} in three-phase fluidized beds.

Table 3. Comparison of Average Absolute Percent Deviations for NU_3 Values of Kato's Correlation with Present Study Correlation

Reference	Kato's* Correlation	Present Correlation	Test Fluid
Baker et al. (1978)	28%	6.14%	Air and water
Kang et al. (1983)	64.5%	10.64%	Air and CMC solution

* Kato et al. (1981).

Heat Transfer in Three-Phase Fluidized Beds

In the previous study, it was found that the modified Stanton number, St_{m3}, is independent of the modified Reynolds number, Re_{m3}, and the relative increase in heat transfer coefficient is equal to the relative decrease in liquid holdup, i.e.,

$$\frac{h_3}{h_2} = \frac{\epsilon_{L2}}{\epsilon_{L3}} \tag{13}$$

or

$$St_{m3} = St_{m2} \tag{14}$$

The values of St_{m3} vs. St_{m2} for the results of Baker et al. (1978), Kato et al. (1981), Kang et al. (1983), and those of the current study are presented in Figure 5. Baker's values agree well with Eq. 14. Those of Kato et al. and Kang et al. agree with Eq. 14 to within ±15%.

Combining Eqs. 9, 10, 12, and 14 leads to

$$Nu_3 = 0.762\, Re_m^{0.646} Pr^{0.638} u_R^{0.266} \phi_s^{-1} \frac{1-\epsilon_{L2}}{\epsilon_{L3}} \tag{15}$$

A comparison of Nu_3 obtained in the present experiments with Nu_3 calculated from Eq. 15 is shown in Figure 6. The values of ϵ_{L3} are equal to those of ϵ_{L2} at zero gas velocity. According to Eqs. 13 and 14, St_{m3} is identical with St_{m2}. Thus, Eq. 15 is a generalized correlation equation for heat transfer coefficient in liquid-solid and three-phase fluidized beds.

Kato et al. (1981) also proposed a correlation equation for the heat transfer coefficients in three-phase fluidized beds. The equation proposed by Kato et al. (1981) is as follows:

$$\frac{h_3 d_p \epsilon_{L3}}{k(1-\epsilon_{L3})} = 0.044 \left(\frac{d_p u_L \rho_L C_{PL}}{k(1-\epsilon_{L3})}\right)^{0.78} + 2\left(\frac{u_G^2}{g d_p}\right)^{0.17} \tag{16}$$

The value of ϵ_{L3} becomes the liquid holdup of the liquid-solid fluidized bed at zero gas velocity.

It was found that the calculated values of heat transfer coefficient with Kato's correlation are considerably higher than those experimentally observed. A comparison of average absolute percent deviations for Nu_3 values of the present correlation with those of Kato's correlation for the results of Baker et al. (1978) and Kang et al. (1983) is made in Table 3.

The effect of gas velocity on the heat transfer coefficient does not appear explicitly in Eqs. 14 and 15. However, ϵ_{L3} and St_{m3}, which includes ϵ_{L3}, which includes ϵ_{L3}, are functions of gas velocity.

The effect of bed temperature on the correlation, Eq. 15, has not been studied in this investigation. Some thermal expansion effects are to be expected, so that liquid holdup in three-phase systems should correspond to liquid holdup in two-phase systems measured or estimated at the same temperature. It should be noted that the data compared with Eq. 15 in this study have all been generated from surface-to-bed heating rather than bed-cooling situations.

CONCLUSIONS

The following conclusions are drawn from the current study:

1. Liquid holdup decreases with increasing gas velocity and decreasing liquid velocity for all particle sizes studied.
2. The liquid holdup values observed in the present work are in agreement with Kato's correlation.
3. The heat transfer coefficient in a liquid-solid fluidized bed passes through a maximum as bed porosity is continuously increased.
4. Heat transfer coefficients in three-phase fluidized beds increase with gas velocity at a given liquid velocity for all particles studied.
5. The values of $h_2\epsilon_{L2}$ are equal to those of $h_3\epsilon_{L3}$.
6. Heat transfer coefficients in both liquid-solid and three-phase fluidized beds are well correlated by Eq. 15 in terms of Nu_3, the modified Reynolds numbers, Re_{m2}, the ratio of minimum fluidization velocity to terminal velocity, the shape factor, and Prandtl number.

NOTATION

C_{PL} = liquid heat capacity, kJ/kg·K
d_p = particle diameter, m
k = liquid thermal conductivity, w/m·K
K = group defined in Eq. 2, dimensionless
h_2 = heat transfer coefficient in liquid-solid fluidized beds, W/m^2·K
h_3 = heat transfer coefficient in three-phase fluidized beds, W/m^2·K
g = acceleration due to gravity m/s^2
L = particle length, m
Nu_2 = Nusselt number in liquid-solid fluidized beds, = h_2d_p/k
Nu_3 = Nusselt number in three-phase fluidized beds, = h_3d_p/k
Pr = Prandtl number, = $C_{PL}\mu_L/k$
Re_{m2} = modified Reynolds number in liquid-solid fluidized beds, defined in Eq. 4
Re_{m3} = modified Reynolds number in three-phase fluidized beds, defined in Eq. 5
S = surface area of particles per unit volume, m^{-1}
St_{m2} = modified Stanton number in liquid-solid fluidized beds, defined in Eq. 6
St_{m3} = modified Stanton number in three-phase fluidized beds, defined in Eq. 7
u_G = superficial gas velocity, m/s
u_L = superficial liquid velocity, m/s
u_R = ratio of minimum fluidization velocity to terminal velocity

Greek Letters

ρ_L = density of liquid, kg/m^3
μ_L = viscosity of liquid, kg/s·m^3
ϵ_3 = bed porosity in three-phase fluidized beds
ϵ_{L2} = liquid holdup in liquid-solid fluidized beds
ϵ_{L3} = liquid holdup in three-phase fluidized beds
ϕ_s = shape factor
σ = liquid surface tension, N/m

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